DERIVATIVES

"A derivative contract is a financial instrument whose payoff structure is derived from the value of the underlying asset".

"A forward contract is an agreement entered today under which one party agrees to buy and the other agrees to sell a specified asset on a specified future date at an agreed price".

"A futures contract is a standardized contract between two parties where one of the parties commits to buy and the other commits to sell, a specified quantity of a specified asset at an agreed price on a given date in the future".

"An option is a contract between two parties under which the buyer of the option buys the right, and not the obligation, to buy (Call option) or sell a (Put option) standardized quantity (contract size) of a financial instrument (underlying asset) at or before a pre-determined date (expiry date) at a price decided in advance (exercise price or strike price)."

Derivative Instruments Snapshot

FEATURE	FORWARD	FUTURES	OPTION
Standardization	No	Yes	Yes
Price Negotiation	Between buyer and seller	Market determined	Option price is market determined. Strike price is exchange determined
Liquidity	No	Yes	Yes
Contract closure	By delivery	By delivery. Or, by paying the price differential. Or, by taking an offsetting position.	By delivery. Or, by paying the price differential. Or, by taking an offsetting position.
Margins	None	Yes	Yes
Guarantor	None	Clearing house	Clearing house
Obligation to perform	Both parties	Both parties	Writer
Profit settlement	End of contract	Daily	Option writer collects premium on T+1

B: PRICING THE FUTURE

Rules of Valuation

The rules of valuation are based on the concepts of "Continuous Compounding" and "Short Selling".

Concept 1: Continuous compounding

In general, $\mathbf{A = P \times (1 + r/m)^{m*n}}$ where, "m" is the number of compounding in a year. For annual compounding m=1, for half yearly compounding m=2, for daily compounding m=365 etc. We can even compound on a continuous basis! In this case, the formula drips down to $A = P \times e^{r*t}$ where "e" is the exponential value, "r" is the rate per annum and "t" is the time in years.

Like compounding, we may run into discounting. Where continuous compounding is involved the present value factor is e^{-rt}. Like present value tables we have the e^{-X} table which gives the value of continuous discounting of Rs.1.

Derivative pricing is done using continuous compounding. In real life, however, people use annual or semiannual compounding. Hence it is necessary to find equivalent rates when the compounding frequencies are different. For example, if a certain rate of interest with half yearly compounding is given, we should find the interest rate which will yield the same amount if continuous compounding is done.

FORMULA 1

Compounding Rates:
Annual Compounding
$A = P \times (1 + r)^n$

Compounding less than a year

$$A = P \times \left(1 + \frac{r}{m}\right)^{mn}$$

Continuous compounding
$A = P \times e^{rt}$
Where,
A = Compounded Value
P = Amount to be compounded.
r = Rate of interest
n = Number of years
e = Exponential value
m = No of compounding in a year.

FORMULA 2

Continuous Discounting:
$A = P \times e^{-rt}$
Where
A = Discounted amount
P = Amount to be discounted
e = Exponential value (- X)
r = Rate of interest
t = No. of years

Concept 2: Short selling

Short selling involves selling a stock which you don't own and buying it back later to square the position. A short seller resorts to this strategy because he expects prices to fall and wants to benefit from the fall. In a falling market this is a good way to make money. Consider this example.

FORMULA 3

Equivalent Rates

Normal to Continuous $r_2 = m \times Ln\left(1 + \frac{r_1}{m}\right)$

Continuous to Normal $r_1 = m \times \left(e^{\frac{r_2}{m}} - 1\right)$

Where, r_1 = Normal rate
r_2 = Continuous compounding rate
m = Frequency of compounding
e = Exponential value (+X)
Ln = Natural logarithm

y = Ln x, then $x = e^y$.

Arbitrage:
FFP = AFP No arbitrage.
FFP ≠ AFP Yes arbitrage.
Where, FFP = Fair forward price or theoretical forward price
AFP = Actual forward price

Pricing of Forward and Futures contracts

Principle 1

Forward contracts will be priced using the cost of carry model and assuming both continuous compounding and the possibilities of short selling. Futures are a close cousin of forwards and hence we will use the phrases interchangeably

Principle 2

Arbitrage opportunities will emerge if the actual forward price (AFP) is not equal to the fair forward price. This is because the investor will buy in one market and simultaneously sell in the other market to make risk free gains.

Rule 1 – Buy spot, Sell forward: If the actual forward price is greater than the fair forward price the stock is overvalued in the forward market. So the investor will borrow money at the risk free rate of interest, buy the stock in the spot market and immediately sell it in the forward market. He then proceeds to pocket the difference.

Rule 2 – Buy forward, Sell spot: If actual forward price is less than the fair forward price the stock is under valued in the forward market. So the investor will sell the stock in the spot market, invest the proceeds at the risk free rate of interest, buy the stock in the forward market and use the maturity proceeds of the investment to settle payment when the forward contract materializes. He then proceeds to pocket the difference.

In general, he will buy in the market where the stock is undervalued and sell in the market where it is overvalued.

Actual FP Vs	Valuatio	Spot	Borrow/Inv	Forward
AFP<FFP	Under	Sell	Invest	Buy
AFP>FFP	Over	Buy	Borrow	Sell

This will apply to any of the forward pricing situations described below.

Situation 1: Securities providing no income

$F = S_0 \times e^{rt}$; where, "F" is the forward price, "S" is the spot price, "r" is the risk free interest rate with continuous compounding and "t" is the time to maturity expressed in years.

Situation 2: Securities providing known cash income

$$F = (S_0 - I) \times e^{rt}$$

Situation 3: Securities providing a known yield

$$F = S_0 \times e^{(r-y) \times t}$$

Situation 4: Carry type commodities

These are commodities that are held for purposes of investment rather than for purposes of consumption. Gold is a typical example. The following points merit attention;

FORMULA 4

Theoretical or Fair Forward price

Securities Providing no income

$F = S_0 \times e^{rt}$

Securities providing known cash income

$F = (S_0 - I) \times e^{rt}$

$I = Y \times e^{-rt}$

Securities providing a known yield

$F = S_0 \times e^{(r-y) \times t}$

Carry type commodity

$F = (S_0 + S) \times e^{rt}$

Non-carry type commodity

$F = (S_0 + S) \times e^{(r-c) \times t}$

Where,

- F = Theoretical forward price
- S_0 = Current spot price
- e = Exponential value
- r = Rate of interest
- t = No of years
- I = Present value of income
- Y = Income
- y = Yield
- S = Present value of storage cost
- c = Convenience yield

- If the storage cost is Nil, this translates into Situation 1, namely securities providing no income. The same formula contemplated in Situation 1 can be adopted.
- If storage cost is involved, the storage cost can be considered as negative cash income. The steps adopted in Situation 2 can be adopted.
- If storage cost is considered as being proportional to the price of the commodity, it can be considered as negative yield. The procedure adopted in Situation 3 can be used.

Situation 5: Non-carry type commodities

These are commodities that are held for purposes of consumption and not for investment. Example: Rice, wheat etc.

The applicable formula will be $F = (S_0 + S) \times e^{((r-c)\ t)}$

Situation 6: Index futures

The Stock Index tracks the changes in a basket of stocks. The value of a Index Futures contract can be ascertained using the cost of carry model.

Here, the spot price is the "Spot Index points", the carry cost is the interest on the value of stock underlying the index, while the "Carry return" is the value of dividends receivable between day of valuation and delivery date. The situation using Known Income or Known Yield, as the case may be, can be applied.

Hedging with Futures Contracts

Cash market, also known as Spot market, is one where the price is agreed on one day and "Delivery and Settlement" is made on the same day. **Derivative market,** also known as Credit market (of which Futures market is a segment) is one where the price is agreed on one day and "Delivery and Settlement" is made on a specified future date.

Principle 3

Hedging is any act which reduces the price risk of a position taken in the cash market. Forward contracts facilitate hedging. In the chapter on International Finance we will learn how this is possible.

What you should do: Futures contract too facilitate hedging. You can hedge with a futures contract by taking a position that is the opposite of the position taken in the cash market. Position can be "Long" or "Short". The term "Long" means bought position. The term "Short" means sold position. Hence:

Rule 1: You should Sell Futures if you have long position on the asset in the cash market. [Short hedge]

Rule 2: You should Buy Futures if you have a short position on the asset in the cash market. [Long hedge]

Spot position	Futures	Price	Spot	Futures
Buy (Go Long)	Sell	Goes up	Gain	Lose
	(Go short)	Falls	Lose	Gain
Sell (Go Short)	Buy	Goes up	Lose	Gain
	(Go long)	Falls	Gain	Lose

The Hedge Ratio

So far we worked on the assumption that the spot market and the futures market move in perfect tandem. This is okay if the underlying asset in the stock market is also traded in the futures market. But if the underlying asset is not traded in the futures market you cannot create a hedge by trading in the futures market (because that asset is not available in that market). However you can use an alternate asset or the Index as hedging tool.

The number of contracts to buy or sell in the Futures market is given by the following formula:

$$\text{Futures Contract} = \frac{\text{Hedge ratio x Rupee value of spot position requiring hedging}}{\text{Rupee value underlying one futures contract}}$$

Note: The rupee value of one unit of NIFTY is Rs.200 and that of one unit of SENSEX is Rs.50.

$$\text{Hedge ratio} = \frac{\sigma_S}{\sigma_F} \text{ x Correlation}_{FS}.$$

where, “S” is the change in spot price and “F” is the change in futures price.

The number of futures contract to trade is given by the formula:

$$\text{Hedge ratio x } \frac{\text{Units of spot position requiring hedging}}{\text{No of units underlying one futures contract}}$$

FORMULA 5

Hedge Ratio

Hedge ratio $= \frac{\sigma_S}{\sigma_F}$ x Correlation$_{FS}$

Where:

S = Change in spot price

F = Change in futures price

Full hedge and partial hedge: We just now learnt that the hedge ratio is really the beta. If we take equivalent opposite position we are able to create full hedge. But sometimes we may not be interested in full hedge. For instance we may be interested only in hedging say to the extent of 75%. What do we do? Simple. Just multiply by 0.75!

Number of future contracts to trade

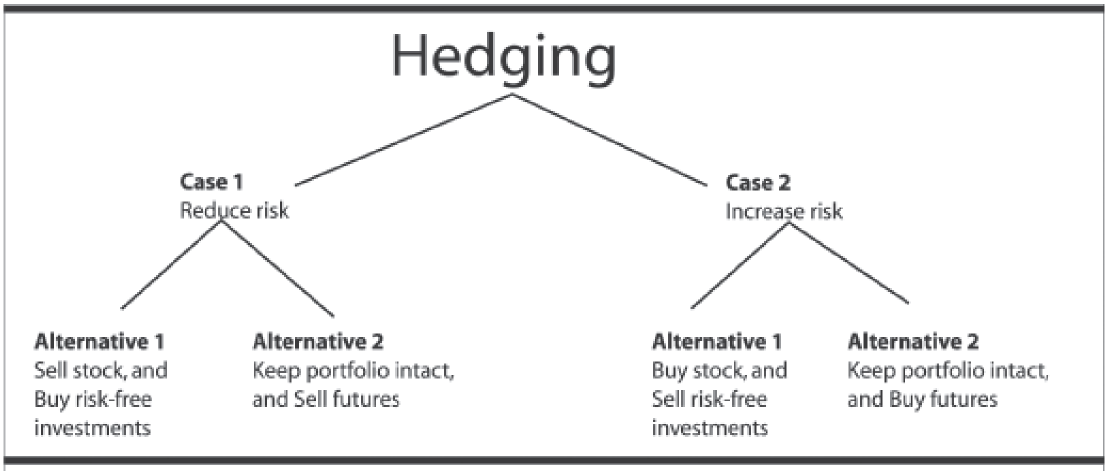
$$\text{Hedge ratio x } \frac{\text{Units in spot position requiring hedging}}{\text{No. of units underlying one future contract}}$$

More on Hedging with Index futures

As a Fund Manager who has just taken over a portfolio or who is currently handling a portfolio you are not happy with the Beta of the portfolio. You want to either increase it or decrease it depending on your attitude to risk. You can do that either by substituting stocks in the portfolio or simply by dealing in Index futures.

$$\text{Futures to be sold} = \text{Hedge ratio x } \frac{\text{Rupee value of spot position requiring hedging}}{\text{Rupee value underlying one futures contract}}$$

Consider this example. A company has three stocks A, B and C which it holds in the proportion 0.50, 0.25 and 0.25. Their respective betas are 1.6, 2.0 and 0.8. The risk of the portfolio is therefore the weighted average beta which in this case is 1.5. This portfolio is riskier than the stock market by 50%. If NIFTY futures is 1700 and are in multiples of 50, we can decrease the portfolio risk to 1.2 or increase it to 1.68. Let's see how.



Case 1: Reduce risk

ALTERNATIVE 1: SELL SOME SECURITIES AND REPLACE WITH RISK FREE INVESTMENT

Step 1: Equate the weighted average beta formula to the new desired beta

He can sell some securities and replace it with risk free investment. We know that the risk free investment has a beta of zero. Suppose the present portfolio is A1 and the risk free investment as A2, the weighted beta of the new portfolio will be

$(Beta_1 \times W_1) + (Beta_2 \times W_2)$

$[Beta_1 \times W_1] + [Beta_2 \times (1-W_1)]$

$1.5 \times W_1 + 0 = 1.2$. Or $W_1 = 0.8$

Step 2: Use the weight and decide

This means that a portfolio of 16 lakhs (0.8 x 20 lakh) invested in the three securities in the proportion given above and the balance Rs.4 lakhs (20 lakhs – 16 lakhs) invested in risk free investments will reduce the beta to 1.2

ALTERNATIVE 2: RETAIN PORTFOLIO IN TACT AND SELL STOCK INDEX FUTURES

Method 1: First Principles

Method 2: Formula Method

$$\frac{(\text{Portfolio Value}) \times (\beta \text{ of the portfolio} - \text{Desired value of } \beta)}{(\text{Value of a futures contract})}$$

Case 2: Increase risk

ALTERNATIVE 1: BUY SOME SECURITIES AND SELL RISK FREE INVESTMENT

Step 1: Equate the weighted average beta formula to the new desired beta.

He can replace risk free investments by buying some securities. We know that the risk free investment has a beta of zero. Suppose the present portfolio is A1 and the risk free investment as A2, the weighted beta of the new portfolio will be

Step 2: Use the weight and decide

ALTERNATIVE 2: RETAIN PORTFOLIO IN TACT AND BUY STOCK INDEX FUTURES

Method 2: Formula Method

$$\frac{(\text{Portfolio Value}) \times (\beta \text{ of the portfolio} - \text{Desired value of } \beta)}{(\text{Value of a futures contract})}$$

C: OPTING FOR OPTIONS

D: TAKE OFF TO STRATEGIES

RULE 1

Party	Increase in Price	Decrease in Price
Call buyer	Favorable	Adverse
Call Writer	Adverse	Favorable
Put buyer	Adverse	Favorable
Put seller	Favorable	Adverse

RULE 2

Option	Right to	EP<MP	EP > MP
Call	Buy	Exercise	Lapse
Put	Sell	Lapse	Exercise

Low down 1: Stock Price movements and value

A call option gives its owner the right to buy a stock at a specified price on or before the expiry date.

An increase in stock price is favourable to the call buyer because he can sell his shares at the higher market price. A drop in price is adverse because it fetches him a lower price and if the price dips below the exercise price he will in fact have to let his option lapse.

The call writer sells the "right to buy"; that is he undertakes the obligation to sell. Hence while any increase in stock price is adverse to him, a reduction in stock price is favourable.

A put buyer buys the right to sell shares. An increase in stock price is adverse since he has bought the right to sell at a lower price and there is no meaning in buying dear and selling cheap. In contrast, a decrease in price is favourable.

The put writer or put seller grants the right to sell; that is he undertakes the obligation to buy. Hence any increase in stock price is favourable and any reduction in stock price is adverse to him.

The table RULE 1 summarizes the position.

Here is a double-quick tool to remember. BUY LOW SELL HIGH. A call gives the buyer the right to buy at Exercise Price. Thereafter, of-course he could sell at market price. Hence in a call the EP is the buying price and the MP is the selling price. If the EP<MP, he would end up buying low and selling high which is good for him. In contrast a put gives the buyer the right to sell at exercise price. Thereafter, of-course he can buy at market price. Hence in a put the EP is the selling price and the MP is the buying price. If EP>MP he ends up selling high and buying low which is good for him. Hence if a buy-sell strategy leads to a gain it is advantageous. The table RULE 2 summarizes the position.

Low Down 2: In-the-money, At-the-money and Out-of-the-money options

An option is said to be "in-the-money" if exercising the option will bring about a gain. An option is said to be "out-of-the-money" if exercising the option will result in a loss. An option is said to be "at-the-money" if exercising the option will result in neither a gain nor a loss.

In this context the option premium paid to buy these options is to be ignored since it represents a sunk cost.

The Exercise Price or Strike price is fixed by the exchange. The exchange announces three prices – one in the money, another at the money and the last out of the money

The table RULE 3 drives home the issue in respect of the various situations for an option buyer.

RULE 3

Relationship	Call Buyer	Put Buyer
Exercise Price > Market Price	Out of the money	In the money
Exercise Price = Market Price	At the money	At the money
Exercise Price < Market Price	In the money	Out of the money

RULE 4

For Buyer	For Writer
OTM: Bad	Good
ATM: Bad	Good
ITM: Good	Bad

The position is expressed only from the standpoint of the Buyer. Thus when an option is In-the-Money it is good for the buyer and bad for the Writer

Low Down 3: Intrinsic Value and Time Value

An option's premium consists of two parts (a) Intrinsic value and (b) Time value. **Intrinsic value** is that part of the option premium which represents the extent to which the option is in the money if it is in the money. This means that in respect of options that are at the money or out of the money there is no intrinsic value. i.e. intrinsic value cannot be negative

Rule 5:

Status	Intrinsic Value	Time value
ITM	If Call: MP – EP If Put: EP – MP	Max (P-IV, 0)
ATM	Nil	Premium
OTM	Nil	Premium

Time Value is the difference between Option Premium and Intrinsic Value and is the premium paid for the time value of money. Time value falls with time and falls to zero on the expiration date. It cannot be negative,

Low Down 4: European Option and American Option

When an option can be exercised on or before the expiry date it is called an **American option**. When an option can be exercised only on the expiry date it is called a **European option**. You must

RULE 6: Exercise

American – Any time
European – Expiry date

know how to spot the nature of the option because the price (i.e. Premium) of an American option will be greater than that of a European option. This is because an American option gives the option holder the right to exercise on any date and not just on the expiry date.

TERM	MEANING
American option	Exercisable any time before the expiry date
European option	Exercisable on expiry date only.
Underlying asset	The asset that can be bought or sold with the option.
Expiry date	Date by which the option has to be exercised.
Option premium	Price to be paid to buy an option.
Buyer	Buys the right to buy or buys the right to sell
Writer	Sells the right
Call option	Buyer gets the right to buy.
Put option	Buyer gets the right to sell.
Exercise price	The price at which the underlying asset will be bought/sold while exercising a call/put

Low Down 5: What are the choices before the option holder and the option writer

After he has **bought an option**, the holder has three choices. The first two choices are available both for American options and for European options. The third choice is available only for American options.

Know it!

Rule 7: Choices

	Holder	Writer
Do nothing	Yes	Yes
Close out	Yes	Yes
Exercise	Yes	No

Choice 1: Do nothing: In this case he sits tight and waits for the expiry date.

Choice 2: Close out: In this case, he does a reverse trade. If he owns a call, he should now write a matching call. If he owns a put he should now write a matching put. [This is analogous to selling his call or his put at the prevailing price of the call or the prevailing price of the put as the case may be].

Choice 3: Exercise the option: In the case of the call option, he will pay the exercise price and receive the shares. In the case of the put option he will deliver the shares and receive the exercise price. It must be remembered that only an American option can be exercised before the expiry date.

After he has **written an option**, the writer has two choices.

Choice 1: Do nothing: In this case he sits tight and waits for the expiry date.

Choice 2: Close out: In this case, he does a reverse trade. If he has written a call, he should now buy a matching call. If he has written a put he should now buy a matching put.

Low Down 6: What happens on the expiry date

A European option cannot be exercised until the expiry date. In the case of an American option if the buyer does not exercise his option until the expiry date, he will have to decide one way or another on the expiry date. In both these cases (European option and unexercised American option), this is what would happen. We explain the logic.

A call buyer buys the right to buy at exercise price and sell at market price. If the exercise price is greater than the market price he would not exercise his option because he will have to buy high and sell low. If the exercise price is less than the market price the call buyer would exercise the option because he can buy low and sell high.

RULE 8

Relationship	Call Buyer	Put Buyer
Exercise Price > Market Price	Lapse	Exercise
Exercise Price = Market Price	Lapse	Lapse
Exercise Price < Market Price	Exercise	Lapse

RULE 9

Party	Gains if	Sentiment
Call Buyer	Price rises	Bullish
Put Buyer	Price falls	Bearish
Call Writer	Price falls	Bearish
Put Writer	Price rises	Bullish

A put buyer buys the right to sell at exercise price and buy at market price. If the exercise price is greater than the market price he would exercise his option because he sells high and buys low. If the exercise price is less than the market price the put buyer would let the option lapse because it is not advantageous to sell low and buy high.

The table RULE 8 explains as to what will happen on the expiry date.

Another way of looking at it is to see whether on the expiry date the option is in-the-money or at-the-money or out-of-the-money. In-the-money options are exercised and the other two are lapsed.

From the table RULE 9 we can infer what are the expectations of the four parties *vis a vis* the underlying asset.

FORMULA 6

Value of an option contract on expiry

Call option $C_1 = \text{Max}(0, S_1 - E)$

Put option $P_1 = \text{Max}(0, E - S_1)$

Where,

C_1 = Value of call on expiry

P_1 = Value of put on expiry

E = Exercise or Strike price

S_1 = Spot price on expiry date

Low down 7: What would be the value of an option on expiry

Issue 1: Call option

The value of a call on the expiry date will depend on whether the stock price on that date will finish above or below the exercise price.

Situation 1: If on the expiry date, the stock price finishes below the exercise price, the call will be out of the money and will not be exercised. Therefore the value of the call will be zero.

Situation 2: If on the expiry date, the stock price is equal to the exercise price, the call will be at the money. At the money calls will not be exercised. Hence, the value of the call will be zero.

Situation 3: If on the expiry date, the stock price finishes above the exercise price, the call will be in the money and will be exercised. The gain will be S1 - E (i.e. Market price less Exercise price). Hence the value of the call will be S1 - E.

Situation 4: A holder will exercise a call option if by buying at EP and selling at MP, he gains. In taking this decision, the premium paid on the option is irrelevant as it represents a sunk cost.

In general, the value of a call is Max (0, S1 - E).

Issue 2: Put option

The value of a put on the expiry date will depend on whether the stock price on that date will finish above or below the exercise price.

Situation 1: If on the expiry date, the stock price finishes below the exercise price, the put will be in the money and will be exercised. Therefore the value of the put will be E-S1.

RULE 10:

Status	Value of Call	Value of Put
EP > MP	Zero	$E-S_1$
EP = MP	Zero	Zero
EP < MP	S_1-E	Zero

Situation 2: If on the expiry date, the stock price is equal to the exercise price, the put will be at the money. At the money puts will not be exercised. Hence, the value of the put will be zero.

Situation 3: If on the expiry date, the stock price finishes above the exercise price, the put will be out of the money and will not be exercised. Hence the value of the put will be zero.

Situation 4: A holder will exercise a put option if by selling at EP and buying at market price, he gains. In taking this decision, premium paid on the option is irrelevant as it represents a Sunk Cost.

In general, the value of the put is Max (0, E - S1).

Notice that for every stock price above the exercise price, the value of the PUT is zero. For every stock price below exercise price the value of a put is E-S1. Once E > S1, the option's value goes up Rupee for rupee with every decrease in stock price.

Remember

A payoff table and a payoff graph will capture succinctly as to how profits or losses are made in each strategy. A key term in preparing the payoff table is the breakeven price. The breakeven price is the stock price at which net payoff is zero. If in the net payoff equation, the stock price is eliminated, there will be no breakeven point.

Low Down 8: Profit graphs or Payoff graph.

The table RULE 11 summarizes the position.

A graph that captures then Net Gain for various anticipated market prices is called a **Payoff Graph**. Such a graph is useful since it offers information in a snapshot. If the graph is drawn without taking into account the premium, it is called a position graph. Initially, it would be a lot easier to understand payoff graphs. As you graduate in your understanding you can draw position graphs.

RULE 11

Party	Gains	Loss
Call Buyer	Unlimited	Limited
Put Buyer	Limited	Limited
Call Writer	Limited	Unlimited
Put Writer	Limited	Limited

Low Down 9: Break-Even price

Break Even Price is the price at which the Net Pay off is zero. It is the market price at which the call buyer or put buyer neither makes a profit nor incurs a loss. Identification of this price is crucial to taking investment decisions.

In terms of equation, it would be

Rule 12

	Call	Put
Buyer	MP – EP – P = 0	EP – MP – P = 0
Seller	EP – MP + P = 0	MP – EP + P = 0

Low down 10: Arbitrage

In Low down 7 we learnt how to arrive at the value of the call option.

If the actual price of the option is not in line with our rules (a k a theoretical price), arbitrage opportunities will open up.

If the actual price is less than the theoretical price, the option is undervalued. If the actual price is greater than the theoretical price, the option is overvalued. Under valued options should be bought and overvalued options should be sold. The following table summarizes the position.

Rule 13

Option type	Status	Action on Option	Action on Stock
Call	Undervalued	Buy	Sell
Call	Overvalued	Sell	Buy
Put	Undervalued	Buy	Buy
Put	Overvalued	Sell	Sell

The under valuation or over valuation is in the derivative market. To make arbitrage gain in the case of a call option, the arbitrageur buys the option in the derivative market (if undervalued) and immediately sells the share in the cash market. Similarly if the call option is overvalued he sells it (i.e. Writes) in the derivative market and buys the share in the cash market.

In the case of an undervalued put option, the arbitrageur will buy a put option and go long in the spot market. In the case of an undervalued put option, he will go short on both the stock and the option.

Low down 11: What would be the value of the option before expiration

This is a trickier question.

All that we can now say is that a call should sell for atleast its intrinsic value. To this would be added the time value, if any. Longer the time to expiry, greater is the time value because you have more time to catch up with the exercise price.

Rule 14

Undervalued if premium is less than intrinsic value

Overvalued if premium is greater than intrinsic value and if no TVM

Similarly all that we can now say is that a put option will usually sell for atleast its intrinsic value. To this would be added the time value. Longer the time to expiry, greater is the time value because you have more time to catch up with the exercise price. What exactly would be the fair price will depend on a string of factors. We will take that up in a section exclusively devoted to valuation where we would take a shot at various valuation models including the Black-Scholes model which won for its authors the Nobel Prize.

Flip it around

The put call parity formula can be turned around in any way like we change the "subject of a formula" in math. For example, this can be rewritten as value of put equals value of call, plus present value of exercise price less value of share. Therefore if put is not available, buy call, Invest present value of exercise price in safe asset and sell share!

Low down 12: What is put-call parity

What is the link between the value of a call and that of a put?

If you do not wish to read the explanations, here's the basic relationship. Namely, "Value of share plus value of put" is equal to "Value of call plus present value of exercise price"!

The formula reads: S + P = C + PV of EP

The formula reads: S + P = C + PV of EP

We can now turn this around nicely to meet our convenience.

For example, P = C + PV of EP - S

Similarly C = S + P - PV of EP

FORMULA 7

Put - Call Parity

S + P = C + PV of Exercise price

Value of call option

C = S + P - PV of Exercise price

Value of put option

P = C + PV of Exercise price - S

Where:

P = Price of a put option

C = Price of a call option

S = Current price of underlying stock

E: STRATEGY

You will now get to see how derivatives can help you make money.

Strategy 2: Spread

The matrix below captures this

Option	Exercise Price Low	Exercise Price High
Call	Higher premium	Lower premium
Put	Lower premium	Higher premium

How to create spreads?

A Bull Spread is created in one of the following two ways

Way 1: Buy a Call at E1 and write a Call at E2

Way 2: Buy a Put at E1 and write a Put at E2

A Bear Spread is created in one of the following 2 ways.

Way 1: Write a Call at E1 and Buy a Call at E2

Way 2: Write a Put at E1 and Buy a Put at E2

In cracking questions on Strategy we adopt three steps.

Step 1: Prepare Relationship Table

Relationship	Option 1	Option 2	GPO	Premium	NPO	BEP
(1)	(2)	(3)	(4)	(5)	(6)	(7)

Column 1: If there are two exercise prices there will be three relationships. The first is market price being less than E1; the second is market price falling between E1 and E2; and the last is market price moving beyond E2. In general if there are "n" exercise prices, there will be "n+1" relationships.

Columns 2 and 3: For the respective options, for respective relationship find out what would be the Gross Pay off.

Column 4: Total payoff = Gross Payoff of Col (2) + Gross Payoff of Col (3)

Column 5: Place the aggregate premium. Have a plus sign if it is premium received and a minus sign if it is net premium paid

Column 6: Column 4 + Column 5

Column 7: Equate Column 6 to zero and find the value of S_1

For each of the relationship we should calculate the aggregate Net Pay off and Break Even arising out of dealings in the options

Step 2: Prepare Break Even Table

Here we need to put in place a class interval and indicate what happens in the class interval. A rough rule of thumb is that the upper limit of class intervals will be exercise price and break even points.

Step 3: Draw Strategy Graph

Given the Break Even Table draw a graph with market price on base axis and profit on vertical axis.

Strategy 3: Butterfly spread

Way 1: Buy 2 calls at mid-strike price. Write one call above and one call below.

Way 2: Write 2 calls at mid-strike price. Buy one call above and one call below.

Way 3: Buy 2 puts at mid-strike price. Write one put above and one put below.

Way 4: Write 2 puts at mid-strike price. Buy one put above and one put below.

A bull butterfly spread would be most profitable if the underlying stock increased in value, and a bear butterfly spread would be most profitable if the underlying stock decreased in value.

Strategy 4: Straddle

1. Straddles involve simultaneous purchase or sale of options with the same strike price and same expiry date.
2. There are two types of straddles - Long and Short
   a. In a long straddle you buy a call and buy a put (same number of calls and same number of puts) at the same exercise price and same expiry date.
   b. In a short straddle you write a call and write a put (same number of calls and same number of puts) at the same exercise price and same expiry date. This is also called straddle write.

Strategy 5: Strips & Straps

1. When an investor expects a huge change in price he might either set up a strip or a strap depending on whether a price fall is more imminent or a price rise.
2. A strip involves buying one call and two puts all with the same exercise price and same expiry date. This is adopted when a decrease in price is more likely than an increase. Since a put is more profitable when a price decrease occurs, two puts are bought.
3. A strap involves buying two calls and one put all with the same exercise price and same expiry date. This is adopted when an increase in price is more likely than a decrease. Since a call is more profitable when a price increase occurs, two calls are bought.

Strategy 6: Strangle

1. A strangle involves the simultaneous purchase or sale of options with same expiry date but with different exercise price.

2. There are two types of strangles - Long and Short
   a. In a long strangle you buy a call and buy a put (same number of calls and same number of puts) at the different exercise price but same expiry date. The exercise price (E1) of the put is lower than the exercise price (E2) call so that a profit will arise if the stock price falls below E1 or raises above E2. Between the two exercise prices neither option will be exercised and there will be a loss equal to the amount of premium paid.
   b. In a short strangle you write a call and write a put (same number of calls and same number of puts) at different exercise price but same expiry date.

Strategy 7: Box spread

A Box spread involves the simultaneous opening of a bull spread and a bear spread on the same underlying asset. A limited profit can be earned if the stock moves in either direction.

Strategy 8: Condors

A condor involves four call options or four put options. The condor can be a long condor or a short condor. A long condor is created by buying calls or by buying puts. A short condor is created by writing calls or by writing puts.

The exercise prices are selected in such a way as to satisfy both the following two equations

- E2 - E1 = E4 - E3
- E3 - E1 = 2 x (E2 - E1)

Case 1: Long condor with calls. Buy calls at E1 and E4. Write calls at E2 and E3.

Case 2: Long condor with puts. Buy puts at E1 and E4. Write puts at E2 and E3.

Case 3: Short condor with calls. Write calls at E1 and E4. Buy calls at E2 and E3.

Case 4: Short condor with puts. Write puts at E1 and E4. Buy puts at E2 and E3.

F: OPTION VALUATION

Model 1: Portfolio Replication Model

Method 1: Constructing Stock equivalent from Risk free investment plus call option:

CASE 1: OPTION FINISHING ONLY IN-THE-MONEY

We summarize the steps involved:

Step 1: Identify whether all options fall only in the money.

Step 2: Compute the risk free investment. This is the present value of exercise price

Step 3: Apply the formula. Co = So - E/(1+Rf).

CASE 2: OPTION FINISHING OUT-OF-THE-MONEY

We summarize the steps involved

Step 1: Compute the option value on expiry date

Step 2: Compute the risk free investment. This is the present value of lower stock price

Step 3: Compute number of calls to be bought using the following formula

$$\text{Calls to be bought} = \frac{\text{Spread in stock prices}}{\text{Spread in call option values}}$$

Step 4: Apply the formula. So = Present Value of Lower Stock Price + Calls bought x Co

Model 2: Constructing option equivalent from common stock

Co = (No of shares to be bought x So) – Amount to be borrowed

No of shares to be bought

$$= \frac{\text{Spread in possible option value}}{\text{Spread in possible share price}}$$

Amount to be borrowed = PV of [(No of shares to be bought x Lower stock price) - Total payoff on downside

Where:

Co = Value of call option today

So = Current spot price of shares

READY TO USE TOOL

Now if you have a problem remembering the steps associated with the two situations, we can normalize them for purposes of problem solving as under

	CASE 1: ONLY ITM	CASE 2: ALSO OTM
Step 1: Option value on Expiry Date	Compute	Compute
Step 2: Risk free investment	Present value of Exercise Price	Present value of Lower Stock Price
Step 3: No. of Calls	$\frac{\text{Spread in Stock Price}}{\text{Spread in Option Value}}$	$\frac{\text{Spread in Stock Price}}{\text{Spread in Option Value}}$
Step 4: Equate to Stock Price, So	Risk free asset + Calls x Call price	Risk free asset + Calls x Call price

Model 2: Constructing Option Equivalents from Common Stock and Borrowing

Here we set up an option equivalent from "common stock and borrowing".

Stock + Borrowing = Call

We summarize in the form of steps

Step 1: Compute the option values on expiry date

Step 2: Compute number of shares to be bought and the amount to be borrowed

a. Number of shares to be bought =

$$\frac{\text{Spread in possible Option Value}}{\text{Spread in possible share prices}}$$

b. Amount to be borrowed = Present value of [(No of Shares to be bought × Lower stock price) – Total payoff on downside arrived in Step 1].

How's that?

1. Stock = Investment + Call

Note: Equation 1 can change to Stock - Investment = Call

Minus Investment = Borrowing

Step 3: Apply the formula

Value of call = Value of shares bought - Bank loan

= [Step 2(a) x Current market price] - Step 2 (b)

Five factors that determine option value

From our formula and the extension relating to volatility we can say that the following five factors determine option values

Factor	Impact on Call
Stock Price	Directly Proportional
Exercise Price	Inversely Proportional
Time to maturity	Directly Proportional
Risk free rate	Directly Proportional
Variance	Directly Proportional

- **The stock price: (So):** The higher the stock price, the greater is the value of the call. That's small wonder because the option gives us the right to buy the stock at a fixed price.
- **Exercise price:** Higher the exercise price, the less is the value of the call. That's because the exercise price is the price we have to pay to get the stock.
- **Time to maturity: (t):** The more the time to maturity, the greater is the value of the call. That's because the stock has more time to climb up.
- **Risk free rate:** Higher the risk free rate, the greater is the value of the call. The logic is simple. The exercise price is a liability. The present value of the liability falls with increase in discount rate.
- **Variance of return on stock:** Higher the variance more is the value of the call. Higher the risk, higher is the value of the call

Model 3: Risk Neutral Model

The risk neutral model is an extension of the Portfolio Replicating model and the Option Replicating model.

If investors are indifferent to risk, we can compute the expected future value of the option and discount it back at the risk free rate to arrive at the current value.

The following would be the steps:

Step 1: Value the calls at the two ends.

Step 2: Compute upside probability and downside probability.

Step 3: The future value of the call is the weighted average of steps 1 and 2.

Step 4: If the value under step 3 is discounted at the risk free rate we would obtain today's value of the call.

Model 4: The Binomial Model

Let's first quickly recap what we have learnt so far in the Valuation models.

There are two situations to the Binomial model. In Situation 1, the period evaluated is a single period. In Situation 2, the period evaluated is multi period chipped and chopped into several periods. The first case can be sorted out with the help of a formula and the second with the help of decision trees.

SITUATION 1: Single Period

The Value of the call is given by the following formula

$$\text{Value of call} = \frac{C_u \frac{[i-d]}{[u-d]} + C_d \frac{[u-i]}{[u-d]}}{i}$$

$$\text{Hedge Ratio} = \frac{C_u - C_d}{S_0\,[u-d]}$$

Where, C_u = Value of call option at upper limit
C_d = Value of call option at lower limit
u = S1/S0 (if S1 > S0)
d = S1/S0 (if (S1 < S0)
i = 1+Rf for the time interval

The Formula

$$C = S_0 \times N[d_1] - E \times e^{-rt}.N[d_2] \quad = \text{Current MP} \times N(d_1) - \text{PV of exercise price} \times N(d_2)$$

$$d_1 = \frac{\text{Ln}\,[S_0/E] + \left[r + 0.5\,\sigma^2\right]\,t}{\sigma\sqrt{t}} \qquad d_2 = \frac{\text{Ln}\,[S_0/E] + \left[r - 0.5\,\sigma^2\right]\,t}{\sigma\sqrt{t}}$$

where,
σ = Standard deviation of continuous compound rate
Ln = Natural log
t = Time remaining before expiration date (Expressed as a fraction of a year)
r = Continuous compound rate risk-free rate of return
S0 = Current market price
E = Exercise price
N = Cumulative area of normal distribution evaluated at d1 and d2

The Assumptions

1. The option is a European option
2. There are no transaction charges
3. There are no taxes
4. The risk free rate is known and is constant over the life of the option
5. The volatility of the underlying asset is known and is constant over the life of the option
6. The underlying asset's continuously compounded rate of return follows a normal distribution
7. The prices of the underlying assets cannot be negative.

G: THE FULL STORY

Our entire discussion thus far centered on European calls. That was Story No 1. We forgot about puts. We forgot about American Calls. We forgot about dividends. We forgot about bonus. We remember them all now. Here we go.

Story 2: European calls with Dividends

We know that the value of a share consists in part the value of dividends. The option holder is not entitled to dividends. Hence in using the formula, you should deduct from the price of the stock the present value of the dividends payable before the option's maturity.

The term "dividend" here refers to income. Not always would the fact that an income is available on an asset be apparent. It is therefore necessary to use a magnifying glass to spot whether the asset holder will receive an income and the option holder will not. For example, when you buy a house, you earn rental income on it. If you buy an option to buy a house you sit out of the income.

Story 3: European puts without Dividends

To value a European put without dividends use the put call parity formula by taking the value of the call as the European call without dividends

Value of put = Value of Call - Value of Stock + PV of Exercise price.

Story 4: European puts with Dividends

To value a European put with dividends use the put call parity formula by taking the value of the call as the European call with dividends

Value of put = Value of Call - Value of Stock + PV of Exercise price.

Story 5: American call without Dividends

An American call can be exercised any time before the expiry date. We know from our formula that in the absence of dividends the value of a call option increases with time to maturity. Hence it does not pay to exercise an American call early.

Look at it from another angle. Suppose you are holding a 3-month American call for which the exercise price is Rs.100. Suppose the current market price is Rs.125. Further suppose that the expiry date is 2 months away.

The option is in the money and it would be tempting to exercise it. If you exercise, you get the stock. 2 months later the stock climbs to Rs.140. That's good for you; but if you had you held the option, you could have exercised it on expiry date and still have the stock worth 140. There was little meaning therefore in exercising it early. If 2 months later the stock nosedives to Rs.110, you lose money. Had you not exercised your option you could have avoided this loss. Either way there was not point exercising the option.

But suppose you exercise the option and immediately dispose off the stock. Well, instead of that you could instead sell option to a third party and profit more since it is likely to be valued at more than its intrinsic value viz., Rs.125-100 =25. In short an American call is unlikely to be exercised early.

Since an American call is unlikely to be exercised early, its value is the same as that of the European call and Black Scholes can be used.

Story 6: American call with Dividends

You just read that an American call without dividends should not be exercised before the expiry date. That way you don't pay the exercise price and can earn interest thereon. Suppose the stock pays dividend. Would the position change?

If the dividend you gain is more than the interest you lose on early exercise, you should exercise. Otherwise, you should hold onto the call.

As we know of now, the only way to crack the value of an American call with dividends is the binomial model. At each stage you must check whether the option is more valuable if exercised just before the ex-dividend date than if held for one more period.

Story 7: American puts without Dividends

Unlike American calls it may sometimes be worthwhile to exercise an American put early. Consider this extreme example. You bought a 3-month put at an exercise price of Rs.15. One month later, today, the stock price hits zero! You must exercise because this is the best you can get as stock prices can't dip below nil.

Of course stock prices seldom hit zero. Hence you must exercise a call when the stock price hits what you consider as the bottom. An American put is always more valuable than a European put.

The Black Scholes model is not applicable for American type options. Binomial model is used.